# Scalar Particle Contribution
# to Higgs Production via Gluon Fusion at NLO

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## Abstract

We consider the gluon fusion production cross section of a scalar Higgs boson in models where fermion and scalar massive colored particles are present. We report analytic expressions for the matrix elements of $gg \to Hg$, $q\bar{q} \to Hg$, and $qg \to Hq$ processes completing the calculation of the NLO QCD corrections in these extended scenarios. The formulas are written in a complete general case, allowing a flexible use for different theoretical models. Applications of our results to two different models are presented: i) a model in which the SM Higgs sector is augmented by a weak doublet scalar in the $SU(N_c)$ adjoint representation. ii) The MSSM, in the limit of neglecting the gluino contribution to the cross section.

*Key words*: Feynman diagrams, Multi-loop calculations, Higgs physics
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# 1 Introduction

The Higgs mechanism for the electroweak symmetry breaking is the still untested part of the Standard Model (SM). The search for the Higgs boson is one of the most important goals of the present experimental program at the Tevatron and, in the near future, at the Large Hadron Collider (LHC) at CERN.

Electroweak (EW) precision physics data and the direct search limit from LEP constrain the possible values of the Higgs mass in the SM quite strongly, with a solid indication that a SM Higgs boson should be lighter than 200 GeV. However, in many extentions of the SM the above bound does not apply because of the presence of new physics (NP) that can affect the EW fit allowing a higher value for the Higgs mass. Theoretical arguments based on perturbative unitarity, triviality and fine-tuning indicate that the crucial mass range to be investigated is up to the TeV scale. The search of the Higgs boson at the LHC has, therefore, to be supported by an accurate theoretical knowledge of the production cross sections, the decay modes, and the important background processes in this range (for a general review see Ref. [1]).

Due to the gluon luminosity, the main production mechanism for a scalar Higgs boson at the LHC is the gluon fusion process ($pp \rightarrow H + X$) [2]. Its cross section in the SM is, all over the range of interesting values of the Higgs mass, one order of magnitude bigger than that of the other main production mechanisms, the Vector Boson Fusion (VBF) [3], and the production in association with heavy quarks and vector bosons [4].

Differently from the other production mechanisms, the gluon fusion is a process that starts at $\mathcal{O}(\alpha_S^2 G_\mu)$, i.e. at the one-loop level. In fact, the Higgs boson does not couple to gluons directly, but only via a loop of colored particles. Thus, the gluon fusion process is the Higgs production mechanism where NP can play the most relevant role changing significantly the value of the production cross section.

The first predictions for the gluon fusion Higgs boson production cross section in the SM dates in the late seventies [2]. More than fifteen years later, in 1991, the calculation of the NLO QCD corrections was completed in the infinite top mass, $m_t$, limit [5] and, successively, retaining the full dependence on the mass of the heavy fermion that runs in the loops [6]. The total effect of the NLO QCD corrections is the increase of the LO cross section by a factor 1.5–1.7, giving a residual renormalization/factorization scale dependence of about 30%. The unexpected size of the NLO QCD radiative corrections, that seem to spoil the validity of perturbation theory, motivated, at the beginning of 2000, the calculation of the NNLO QCD corrections, performed in the infinite $m_t$ limit [8]. The calculation shows a good convergence of the perturbative series. The NNLO corrections are sizable, but, nevertheless, smaller that the NLO ones. Moreover, the QCD bands of variation of the renormalization/factorization scale overlap with the ones of the NLO calculation. The NNLO corrections enhance the cross section of an additional 15% (of the NLO results). Moreover, they improve the stability against renormalization/factorization scale variations. Furthermore, the effect due to the resummation of soft-gluon radiation at the NNLL accuracy has been evaluated in Ref. [9]. Besides an additional enhancement of the cross section of the order of some percents (up to 6%), the effect mainly lies in a strong reduction of the scale dependence. The remaining theoretical uncertainty, due to higher-order QCD corrections, has been estimated to be smaller than 10%. This estimate was confirmed recently

by the NNNLO calculation of Ref. [10].

Because of the high accuracy reached in the evaluation of the QCD corrections, also the EW NLO corrections to gluon fusion were recently taken into account. In Ref. [11], they were evaluated in the infinite $m_t$ limit, giving a correction of less than 1%. In Ref. [12], the contributions coming from Feynman diagrams with a closed loop of light fermions were calculated in a closed analytic form, expressing the formulas in terms of generalized harmonic polylogarithms (GHPLs) [13]. It turned out that they are sizeable. In particular, in the intermediate Higgs mass range, from 114 GeV up the the $2\,m_W$ threshold, these corrections increase the LO cross section by an amount of 4 to 9%. For $m_H > 2\,m_W$, they change sign and reduce the LO cross section; however, in this region the light-fermion corrections are quite small, reaching at most a -2%. In Ref. [14], also the remaining EW corrections due to the top quark were calculated as a Taylor expansion in $m_H^2/(4m_W^2)$. This result is valid in the $m_H \leq 2\,m_W$ range in which the corrections due to the top quark have opposite sign with respect to the light-fermion contribution. However, the former are smaller in size, reaching at most a 15% of the latter.

Several efforts have also been devoted to the calculation of radiative corrections (mainly QCD) to less inclusive quantities, like the transverse momentum ($q_T$) distribution [15, 16, 17, 18, 19] and the rapidity distribution [20, 21, 19]. All these results have been implemented in two Monte Carlos that calculate fully-differential distributions at the NNLO [22, 23].

The Higgs boson gluon fusion production cross section was also extensively studied in the Minimal Supersymmetric Standard Model (MSSM). The effects of the squarks on the production cross section for the neutral CP-even $h$ and $H$ Higgs bosons were considered in Ref. [24], including the NLO QCD corrections evaluated in the heavy squark mass limit. This approximation has been relaxed in Ref. [25] where the full dependence on the squarks masses has been retained. The complete MSSM NLO QCD corrections to the Higgs bosons production, in the heavy SUSY particles limit, has been presented in Ref. [26]. In general, these corrections lead to a NLO $K$ factor that differs from the corresponding SM one by an amount less than 5%, with the exception of regions where the squark and quark contributions interfere negatively giving rise to a MSSM production cross section much smaller than the SM one [27]. Not inclusive quantities were also studied in the MSMM [28].

As already pointed out, the Higgs boson production via gluon fusion, as well as the Higgs decay into two photons, are processes sensitive to any kind of NP. Thus, it would be desirable to have predictions for these quantities, at the level of NLO QCD corrections, provided in full generality, i.e. not constrained by a particular theoretical model but flexible enough to be used for different models, and, if possible, expressed in an analytic form easy to be evaluated numerically.

The aim of this paper is to analyze in a, as much as possible, model independent way the contribution of colored scalar particles to the gluon fusion Higgs boson production cross section at the NLO level in the QCD corrections. In this spirit, we present here general analytic formulas for the NLO corrections to the production cross section of a Higgs boson via gluon fusion, $\sigma(pp \to H + X)$. In particular, we provide analytic expressions for the NLO QCD corrections to the partonic processes $gg \to Hg$, $q\bar{q} \to Hg$, and $qg \to Hq$, in the two cases in which a fermion or a scalar run in the loops. Together with the analytic results of Refs. [29, 30], where the two-loop virtual QCD correction to the gluon fusion process where evaluated, the present work completes the NLO calculation of the Higgs production

cross section in the presence of colored scalar particles.

The paper is organized as follows. In Section 2, we provide all the analytic formulas and we discuss the validity of the heavy fermion/scalar mass limit. Applications of our results to two different models are presented in the following section. In Section 3.1 we consider a model, proposed by Manohar and Wise [31], in which the Standard Model is supplemented by an additional scalar colored isospin doublet. We discuss, at the NLO level, the effects of these additional scalar particles on the production cross section of the standard Higgs boson and on its decay width $H \to \gamma\gamma$. In Section 3.2, we consider the contributions due to the squarks in the MSSM. We focus on the NLO QCD corrections (exchange of gluons), neglecting the contributions coming from the gluino. Finally, in Section 4 we present our conclusions.

## 2 Higgs Production via Gluon Fusion at NLO

In this section we present analytic results for the NLO QCD corrections to Higgs boson production via the gluon fusion mechanism. Being the Higgs boson neutral under $SU(N_c)$, its coupling to the gluons is mediated by a loop of colored particles. To discuss the gluon fusion mechanism in a general way we assume as colored particles one fermion and one scalar in a generic $R_{1/2}$, $R_0$ $SU(N_c)$ representation, respectively. The extension to more fermions or scalars is trivial. The coupling's strengths of these particles to the Higgs are assumed to be:

$$HFF = g\,\lambda_{1/2}\,\frac{m_{1/2}}{2\,m_W}, \qquad HSS = g\,\lambda_0\,\frac{A^2}{m_W}\,, \tag{1}$$

where $g$ is the $SU(2)$ coupling, $m_W$ is the W mass, $m_{1/2}$ is the fermion mass, $A$ is a generic coupling with the dimension of mass and $\lambda_i$ are numerical coefficients[1].

The hadronic cross section for the Higgs production via gluon fusion at center-of-mass energy $\sqrt{s}$, can be written as:

$$\sigma(h_1 + h_2 \to H + X) = \sum_{a,b} \int_0^1 dx_1 dx_2\, f_{a,h_1}(x_1, \mu_F^2)\, f_{b,h_2}(x_2, \mu_F^2) \times$$

$$\times \int_0^1 dz\, \delta\left(z - \frac{\tau_H}{x_1 x_2}\right) \hat{\sigma}_{ab}(z)\,, \tag{2}$$

where $\tau_H = m_H^2/s$, $\mu_F$ is the factorization scale, $f_{a,h_i}(x, \mu_F^2)$, the parton density of the colliding hadron $h_i$ for the parton of type $a$, $(a = g, q, \bar{q})$ and $\hat{\sigma}_{ab}$ the cross section for the partonic subprocess $ab \to H + X$ at the center-of-mass energy $\hat{s} = x_1 x_2 s = m_H^2/z$. The latter can be written as:

$$\hat{\sigma}_{ab}(z) = \sigma^{(0)}\, z\, G_{ab}(z)\,, \tag{3}$$

where

$$\sigma^{(0)} = \frac{G_\mu \alpha_s^2(\mu_R^2)}{128\,\sqrt{2}\,\pi} \left| \sum_{i=0,1/2} \lambda_i \left(\frac{A^2}{m_0^2}\right)^{1-2i} T(R_i)\,\mathcal{G}_i^{(1l)} \right|^2 \tag{4}$$

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[1]The SM is recovered with $\lambda_{1/2} = 1$, $\lambda_0 = 0$, $N_c = 3$ and $R_{1/2} = \mathbf{3}$.

is the Born-level contribution with $m_0$ the mass of the scalar particle,

$$
\begin{align}
\mathcal{G}_{1/2}^{(1l)} &= -4y_{1/2}\left[2 - \left(1 - 4y_{1/2}\right) H(0,0,x_{1/2})\right], \tag{5}\\
\mathcal{G}_{0}^{(1l)} &= 4y_0\left[1 + 2\,y_0\, H(0,0,x_0)\right] \tag{6}
\end{align}
$$

and $T(R_i)$ is the matrix normalization factor of the $R_i$ representation ($T(R) = 1/2$ for the fundamental representation of $SU(N_c)$, $T(R) = N_c$ for the adjoint one). In Eqs. (5-6)

$$
y_i \equiv \frac{m_i^2}{m_H^2}, \qquad x_i \equiv \frac{\sqrt{1 - 4y_i} - 1}{\sqrt{1 - 4y_i} + 1} \qquad i = 0, 1/2\,, \tag{7}
$$

and, employing the standard notation for the Harmonic Polylogarithms (HPLs), $H(0,0,z)$ labels a HPL of weight 2 that results to be[2]

$$
H(0,0,z) = \frac{1}{2}\log^2(z)\,. \tag{8}
$$

Up to NLO terms, we can write

$$
G_{ab}(z) = G_{ab}^{(0)}(z) + \frac{\alpha_s(\mu_R^2)}{\pi}\, G_{ab}^{(1)}(z)\,, \tag{9}
$$

with

$$
\begin{align}
G_{ab}^{(0)}(z) &= \delta(1-z)\,\delta_{ag}\,\delta_{bg}\,, \tag{10}\\
G_{gg}^{(1)}(z) &= \delta(1-z)\left[C_A\frac{\pi^2}{3} + \beta_0\ln\left(\frac{\mu_R^2}{\mu_F^2}\right) + \sum_{i=0,1/2}\mathcal{G}_i^{(2l)}\right]\notag\\
&\quad + P_{gg}(z)\ln\left(\frac{\hat{s}}{\mu_F^2}\right) + C_A\frac{4}{z}(1 - z + z^2)^2\,\mathcal{D}_1(z) + C_A\,\mathcal{R}_{gg}\,, \tag{11}\\
G_{q\bar{q}}^{(1)}(z) &= \mathcal{R}_{q\bar{q}}\,, \tag{12}\\
G_{qg}^{(1)}(z) &= P_{gq}(z)\left[\ln(1-z) + \frac{1}{2}\ln\left(\frac{\hat{s}}{\mu_F^2}\right)\right] + \mathcal{R}_{qg}\,. \tag{13}
\end{align}
$$

We discuss the various NLO contributions.

i) The $gg$ channel (Eq. (11)) involves virtual and real corrections. The former, regularized by the infrared singular part of the cross section $gg \to Hg$, are displayed in the first row of Eq. (11) where $\beta_0 = (11\,C_A - 4\,n_f\,T(R_f) - n_s\,T(R_s))/6$ with $n_f$ ($n_s$) the number of active fermion (scalar) flavor in the representation $R_f$ ($R_s$). The functions $\mathcal{G}_i^{(2l)}$ containing the mass-dependent contribution of the two-loop virtual corrections, can be cast in the following form:

$$
\begin{align}
\mathcal{G}_i^{(2l)} &= \lambda_i\left(\frac{A^2}{m_0^2}\right)^{1-2i} T(R_i)\left(C(R_i)\,\mathcal{G}_i^{(2l,C_R)}(x_i) + C_A\,\mathcal{G}_i^{(2l,C_A)}(x_i)\right)\notag\\
&\quad \times \left(\sum_{j=0,1/2}\lambda_j\left(\frac{A^2}{m_0^2}\right)^{1-2j} T(R_j)\,\mathcal{G}_j^{(1l)}\right)^{-1} + h.c. \tag{14}
\end{align}
$$

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[2]All the analytic continuations are obtained with the replacement $-m_H^2 \to -m_H^2 - i\epsilon$

where $C(R_i)$ is the Casimir factor of the $R_i$ representation (in particular, for the fundamental and the adjoint representations of $SU(N_c)$ we have $C_F = (N_c^2 - 1)/(2N_c)$ and $C_A = N_c$, respectively). Explicit analytic expressions for $\mathcal{G}_i^{(2l)}$ ($i = 0, 1/2$) given in terms of HPLs can be found in Ref. [29]. It should be noticed that $\mathcal{G}_i^{(2l,C_R)}$ depend upon the choice of the renormalized parameters (masses and couplings). In Ref. [29] expressions for $\mathcal{G}_i^{(2l,C_R)}$ with $\overline{\text{MS}}$ or on-shell (OS) parameters are presented. In the case of single heavy fermion, i.e. $\lambda_0 = 0$, $\mathcal{G}_{1/2}^{(2l)}$ become independent of the renormalized mass chosen and goes to the well know result $-3/2\,C(R_{1/2}) + 5/2\,C_A$, that can be also obtained via an effective theory calculation [5]. The case of a single heavy scalar ($\lambda_{1/2} = 0$) is actually more complicated because, in general, the coupling of a colored scalar particle to the Higgs boson is not directly proportional to the mass of the scalar and therefore different renormalization prescriptions for $A$ and $m_0$ can be employed. In case an $\overline{\text{MS}}$ prescription is employed both for $A$ and $m_0$, the function $\mathcal{G}_0^{(2l)}$ tends, for large values of $m_0$, to the constant value $9/2\,C(R_0) + C_A$ independent on the $\overline{\text{MS}}$ subtraction scale.

The second row of Eq. (11) contains the non-singular contribution from the real gluon emission in the gluon fusion process where

$$\mathcal{D}_i(z) = \left[\frac{\ln^i(1-z)}{1-z}\right]_+ \tag{15}$$

are the plus distributions and

$$P_{gg}(z) = 2\,C_A\left[\mathcal{D}_0(z) + \frac{1}{z} - 2 + z(1-z)\right] \tag{16}$$

is the LO Altarelli-Parisi splitting function. The function $\mathcal{R}_{gg}$ can be written as

$$\mathcal{R}_{gg} = \frac{1}{z(1-z)}\int_0^1 \frac{dv}{v(1-v)}\left\{\frac{8\,z^4\left|\mathcal{A}_{gg}(\hat{s},\hat{t},\hat{u})\right|^2}{\left|\sum_{j=0,1/2}\lambda_j\left(\frac{A^2}{m_0^2}\right)^{1-2j}T(R_j)\,\mathcal{G}_j^{(1l)}\right|^2} - (1-z+z^2)^2\right\}, \tag{17}$$

where $\hat{t} = -\hat{s}(1-z)(1-v)$, $\hat{u} = -\hat{s}(1-z)v$, with

$$\left|\mathcal{A}_{gg}(s,t,u)\right|^2 = |A_2(s,t,u)|^2 + |A_2(u,s,t)|^2 + |A_2(t,u,s)|^2 + |A_4(s,t,u)|^2. \tag{18}$$

Furthermore, the functions $A_2$ and $A_4$ can be cast in the following form:

$$A_2(s,t,u) = \sum_{i=0,1/2}\lambda_i\left(\frac{A^2}{m_0^2}\right)^{1-2i}T(R_i)\,y_i^2\left[b_i(s_i,t_i,u_i) + b_i(s_i,u_i,t_i)\right], \tag{19}$$

$$A_4(s,t,u) = \sum_{i=0,1/2}\lambda_i\left(\frac{A^2}{m_0^2}\right)^{1-2i}T(R_i)\,y_i^2\left[c_i(s_i,t_i,u_i) + c_i(t_i,u_i,s_i)\right.$$
$$\left. + c_i(u_i,s_i,t_i)\right], \tag{20}$$

with

$$s_i \equiv \frac{s}{m_i^2}, \qquad t_i \equiv \frac{t}{m_i^2}, \qquad u_i \equiv \frac{u}{m_i^2}. \tag{21}$$

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We find

$$b_{1/2}(s,t,u) = B_{1/2}(s,t,u) + \frac{s}{4}\left[H(0,0,x_{1/2}) - H(0,0,x_s)\right]$$
$$- \left(\frac{s}{2} - \frac{s^2}{s+u}\right)\left[H(0,0,x_{1/2}) - H(0,0,x_t)\right]$$
$$- \frac{s}{8}H_3(s,u,t) + \frac{s}{4}H_3(t,s,u), \tag{22}$$

$$b_0(s,t,u) = -\frac{1}{2}B_0(s,t,u), \tag{23}$$

$$c_{1/2}(s,t,u) = C_{1/2}(s,t,u) + \frac{1}{2\,y_{1/2}}\left[H(0,0,x_{1/2}) - H(0,0,x_s)\right] + \frac{1}{4\,y_{1/2}}H_3(s,u,t), \tag{24}$$

$$c_0(s,t,u) = -\frac{1}{2}C_0(s,t,u), \tag{25}$$

where

$$x_a \equiv \frac{\sqrt{1-4/a}-1}{\sqrt{1-4/a}+1} \qquad (a = s,t,u)$$

and

$$B_i(s,t,u) = \frac{s(t-s)}{s+t} + \frac{2\,(tu^2+2stu)}{(s+u)^2}\left[\sqrt{1-4y_i}\,H(0,x_i) - \sqrt{1-4/t}\,H(0,x_t)\right]$$
$$- \left(1 + \frac{tu}{s}\right)H(0,0,x_i) + H(0,0,x_s)$$
$$- 2\left(\frac{2s^2}{(s+u)^2} - 1 - \frac{tu}{s}\right)\left[H(0,0,x_i) - H(0,0,x_t)\right]$$
$$+ \frac{1}{2}\left(\frac{tu}{s} + 3\right)H_3(s,u,t) - H_3(t,s,u), \tag{26}$$

$$C_i(s,t,u) = -2s - 2\left[H(0,0,x_i) - H(0,0,x_s)\right] - H_3(u,s,t). \tag{27}$$

In Eqs. (22–27) $H(0,x) \equiv \ln(x)$, and the function $H_3$ is symmetric under the interchange of its last two arguments, i.e. $H_3(a,b,c) = H_3(a,c,b)$, as can be seen from its integral representation:

$$H_3(a,b,c) = \int_0^1 dx \frac{1}{x(1-x)+a/(b\,c)}\{\ln[1 - bx(1-x)] + \ln[1 - cx(1-x)]$$
$$- \ln[1 - (a+b+c)x(1-x)]\}. \tag{28}$$

An explicit analytic expression for $H_3(a,b,c)$ can be found, for instance, in Ref. [15]. Using the notations of [15], we have $H_3(a,b,c) = -W_3(b,a,c,a+b+c)^3$.

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[3]In Ref. [15], in the function $W_3$ the mass of the heavy particle is explicitely written in the integral representation. Instead the $H_3$ function has as input parameters "reduced" variables, i.e. Mandelstam variables divided by the heavy particle mass. Therefore, in order to have the correct formal expression for $H_3$ one has to put in the formulas of Ref. [15] $m_f = 1$ and consider the variables $s$, $t$ and $u$ as reduced variables.

In the case of a single heavy fermion or a single heavy scalar or both fermion and scalar heavy $\mathcal{R}_{gg} \to -11(1-z)^3/(6z)$.

ii) The $q\bar{q} \to Hg$ annihilation channel, Eq. (12), can be written as

$$\mathcal{R}_{q\bar{q}} = \frac{128}{27} \frac{z\,(1-z)\,\left|\mathcal{A}_{q\bar{q}}(\hat{s},\hat{t},\hat{u})\right|^2}{\left|\sum_{j=0,1/2}\lambda_j\left(\frac{A^2}{m_0^2}\right)^{1-2j}T(R_j)\,\mathcal{G}_j^{(1l)}\right|^2}\,,\tag{29}$$

with

$$\mathcal{A}_{q\bar{q}}(s,t,u) = \sum_{i=0,1/2}\lambda_i\left(\frac{A^2}{m_0^2}\right)^{1-2i}T(R_i)\,y_i\,d_i(s_i,t_i,u_i)\,.\tag{30}$$

We find

$$d_{1/2}(s,t,u) = D_{1/2}(s,t,u) - 2\left[H(0,0,x_{1/2}) - H(0,0,x_s)\right]\,,\tag{31}$$

$$d_0(s,t,u) = -\frac{1}{2}D_0(s,t,u),\tag{32}$$

with

$$D_i(s,t,u) = 4 + \frac{4\,s}{(t+u)}\left[\sqrt{1-4y_i}\,H(0,x_i) - \sqrt{1-4/s}\,H(0,x_s)\right]$$

$$+\frac{8}{t+u}\left[H(0,0,x_i) - H(0,0,x_s)\right].\tag{33}$$

In the case of a single heavy fermion or a single heavy scalar or both fermion and scalar heavy $\mathcal{R}_{q\bar{q}} \to 32(1-z)^3/(27z)$.

iii) Finally we consider the quark-gluon scattering channel, $qg \to qH$. In Eq. (13) $P_{gq}$ is the LO Altarelli-Parisi splitting function

$$P_{gq}(z) = C_F\,\frac{1+(1-z)^2}{z}\,,\tag{34}$$

while the function $\mathcal{R}_{qg}$ can be written as

$$\mathcal{R}_{qg} = C_F\int_0^1\frac{dv}{(1-v)}\left\{\frac{1+(1-z)^2v^2}{[1-(1-z)v]^2}\frac{2\,z\left|\mathcal{A}_{qg}(\hat{s},\hat{t},\hat{u})\right|^2}{\left|\sum_{j=0,1/2}\lambda_j\left(\frac{A^2}{m_0^2}\right)^{1-2j}T(R_j)\,\mathcal{G}_j^{(1l)}\right|^2} - \frac{1+(1-z)^2}{2z}\right\}$$

$$+\frac{1}{2}C_F z\,,\tag{35}$$

where

$$\mathcal{A}_{qg}(\hat{s},\hat{t},\hat{u}) = \mathcal{A}_{qq}(\hat{t},\hat{s},\hat{u}).\tag{36}$$

In the case of a single heavy fermion or a single heavy scalar or both fermion and scalar heavy $\mathcal{R}_{qg} \to 2\,z/3 - (1-z)^2/z$.
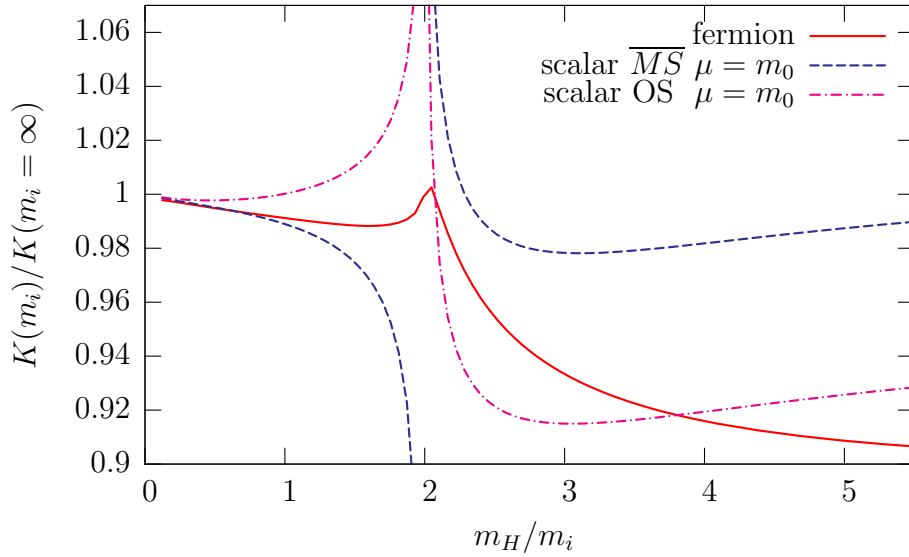
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Figure 1: Exact K-factor normalized to the effective one. The different lines represent a single heavy fermion (solid line) and a single heavy scalar whose mass is renormalized in the $\overline{\text{MS}}$ (dashed line) or in the on-shell scheme (dashed-dotted line).

The expressions reported above are, for the fermionic case, in full agreement with the known results in the literature [15, 16]. They give the exact, i.e. for any value of the particle masses, NLO contribution and correspondingly the exact K-factor defined as the ratio between the NLO and LO cross sections. It is interesting to compare the value of the exact K-factor with the one that can be obtained via an improved effective theory calculation. By the latter we mean a result in which the effective NLO cross section is obtained by multiplying the exact LO cross section by the $\mathcal{O}(\alpha_s)$ corrections evaluated in the heavy particle limit [7]. As discussed above, while for fermions the NLO contribution in the limit of heavy mass is independent upon the definition of the renormalized mass used, the heavy scalar NLO contribution is actually dependent on the renormalization conditions chosen.

In Fig. 1 we plot the exact NLO K-factor normalized to the effective one as a function of $m_H/m_i$, for the case of a single fermion (continuos line) and of a single scalar. For the latter we consider two options: i) On-shell condition for $m_0$ and $\overline{\text{MS}}$ renormalization of the coupling $A$ defined at the $\overline{\text{MS}}$ $\mu$ scale $\mu = m_0$, with $A(m_0) = m_0$ (dash-dotted line). ii) $\overline{\text{MS}}$ renormalization both for $A$ and $m_0$ with $\mu = m_0$ and $A(m_0) = m_0(\mu)$ (dashed line). The results presented in the figure have been obtained assuming a hadronic center-of-mass energy $\sqrt{s} = 14$ TeV, using the parametrization CTEQ6M [34] to describe the partonic content of the proton, and setting the factorization and renormalization scales equal to the Higgs boson mass. These choices will be used also in the figures of the following sections. From Fig. 1 it appears that in the fermion case the difference between the exact and the effective K-factor is at most 10% and that already when $m_H < 2\,m_{1/2}$ the difference is below 1%. Instead, in the scalar case the situation is more complicated. Both cases i) and ii) show a spike at the opening of the $m_H = 2\,m_0$ threshold. This spike is due to logarithmic

and square root singularities present in the two-loop one-particle irreducible (1PI) virtual corrections, coming from diagrams in which a scalar self-energy diagram is inserted in a one-loop vertex diagram. When the mass of the scalar is renormalized on-shell, the mass-counterterm diagrams show a square root singularity that actually cancels the similar one coming from the 1PI diagrams, leaving only an unphysical logarithmic singularity (see dash-dotted line in Fig. 1) related to the inadequateness of the standard mass renormalization procedure in case of unstable particles. Instead, in the case an $\overline{\text{MS}}$ renormalization for the mass of the scalar is employed, the cancellation of the square root singularity between 1PI diagrams and mass-counterterm diagrams does not take place anymore. Then both the square root and the logarithmic unphysical singularities are left (see dashed line in Fig. 1). In the region away from the threshold, the figure shows a good convergence to 1 for light Higgs masses. For a heavy Higgs, instead, we note a certain deviation of the effective theory from the exact one. Nevertheless, this deviation remains quite limited in size, and it reaches at most 9%.

# 3 Scalar Particle Effects on the Higgs Production Cross Section

In this section we discuss the effect of colored scalar particles on the Higgs production cross section via gluon fusion. We consider two cases: i) a model in which the SM fields are supplemented by a weak doublet of colored scalars. ii) The squark contribution in the MSSM.

## 3.1 The Manohar-Wise Model

The model proposed by Manohar and Wise (MW) [31] is an extension of the SM that includes additional colored scalar fields which transform in the $(\mathbf{8}, \mathbf{2})_{1/2}$ representation of $SU(3) \times SU(2) \times U(1)$. The choice of these additional scalar fields is dictated by requirement of natural suppression of flavor changing neutral currents. The additional colored scalar weak doublet

$$S^a = \begin{pmatrix} S_+^a \\ S_0^a \end{pmatrix} = \begin{pmatrix} S_+^a \\ \frac{S_{0R}^a + iS_{0I}^a}{\sqrt{2}} \end{pmatrix} \tag{37}$$

with $a = 1 \dots 8$ an adjoint color index, contains an electrically charged and two neutral real scalars. Denoting the standard $(\mathbf{1}, \mathbf{2})_{1/2}$ Higgs field by $H$, the most general potential can be written as [31]

$$
\begin{aligned}
V &= \frac{\lambda}{4} \left( H^{\dagger i} H_i - \frac{v^2}{2} \right)^2 + 2m_S^2 \, Tr S^{\dagger i} S_i \\
&+ \lambda_1 H^{\dagger i} H_i Tr S^{\dagger j} S_j + \lambda_2 H^{\dagger i} H_j Tr S^{\dagger j} S_i + \left( \lambda_3 H^{\dagger i} H^{\dagger j} Tr S_i S_j + h.c. \right) + \cdots
\end{aligned} \tag{38}
$$

where $S = S^a T^a$, the trace is over color and $SU(2)$ indices are explicitly shown. The ellipses represent tri- and quadrilinear interaction terms of the fields $S^a$ not relevant for our
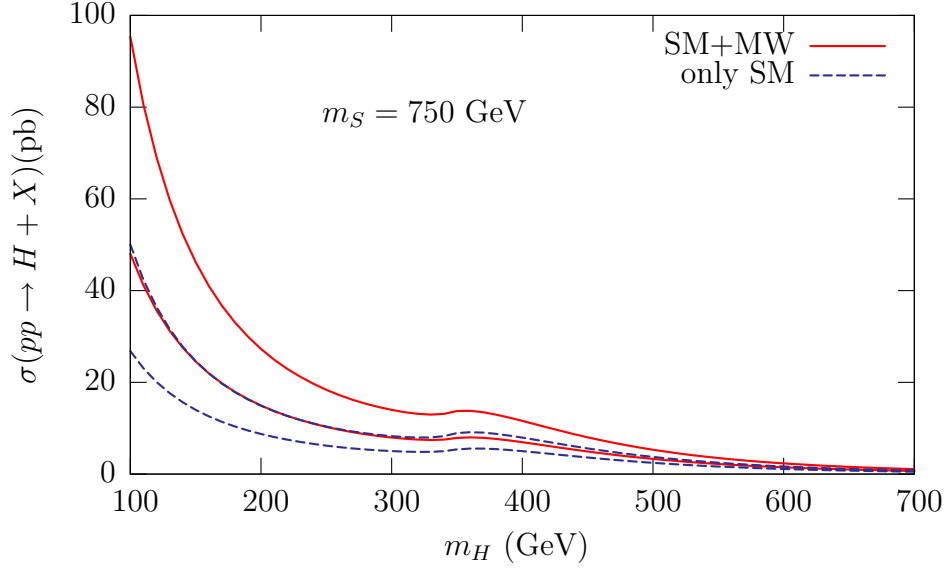
Figure 2: Higgs production cross sections as a function of the Higgs mass. The particles running in the loop are: only SM fermions (dashed lines) or both SM fermions and MW scalars (solid lines). The lower curves in the two cases represent the LO cross section, while the upper curves represent the NLO one. The couplings are $\lambda_1(m_S) = 4$, $\lambda_2(m_S) = 1$, $\lambda_3(m_S) = 1/2$. The scalar mass is $m_S = 750$ GeV.

discussion. The tree-level mass spectrum of the colored octet scalars is found to be:

$$
\begin{aligned}
m_{S_+}^2 &= m_S^2 + \lambda_1 \, \frac{v^2}{4}, \\
m_{S_{0R}}^2 &= m_S^2 + (\lambda_1 + \lambda_2 + 2\lambda_3) \, \frac{v^2}{4}, \\
m_{S_{0I}}^2 &= m_S^2 + (\lambda_1 + \lambda_2 - 2\lambda_3) \, \frac{v^2}{4}.
\end{aligned}
\tag{39}
$$

and the coupling to the standard Higgs are:

$$
\begin{aligned}
HS_+^a S_-^b &= g \, \frac{\lambda_1}{4} \frac{v^2}{m_W} \delta^{ab}, \\
HS_{0R}^a S_{OR}^b &= g \frac{\lambda_1 + \lambda_2 + 2\lambda_3}{8} \, \frac{v^2}{m_W} \delta^{ab}, \\
HS_{0I}^a S_{0I}^b &= g \frac{\lambda_1 + \lambda_2 - 2\lambda_3}{8} \, \frac{v^2}{m_W} \delta^{ab}.
\end{aligned}
\tag{40}
$$

Eqs. (39,40) show that in this model $A/m_0 \sim v/m_S$ for $m_S \gg v$ ensuring the decoupling of the colored scalars as their mass increases.

To analyze the effect on the Higgs production cross section of this octet of colored scalars we plot, in Fig. 2, the LO and NLO cross sections, as a function of $m_H$, including the scalar contribution and compare them with the SM results. The figure is drawn taking
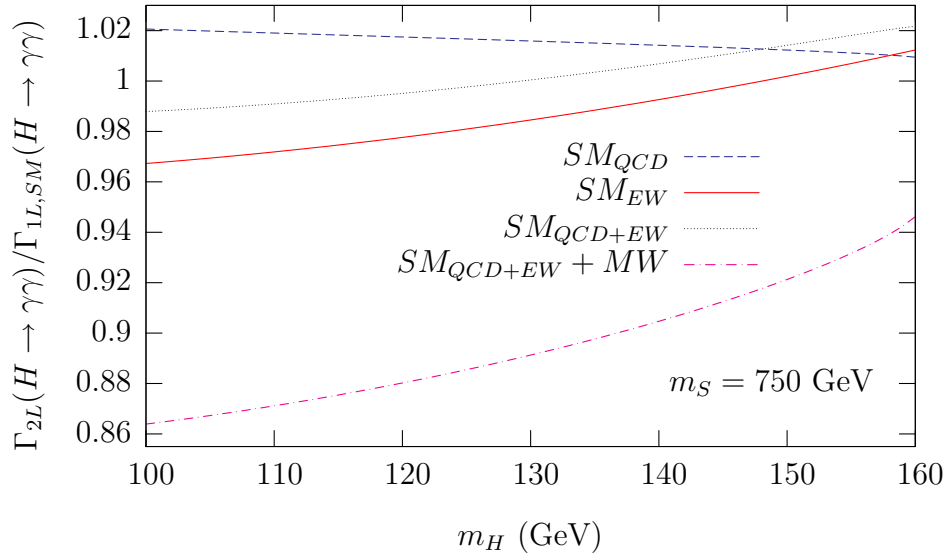
Figure 3: NLO decay width, $\Gamma_{2L}(H \to \gamma\gamma)$ in the SM and in MW model, normalized to the LO decay width $\Gamma_{1L,SM}(H \to \gamma\gamma)$ in the SM. The dashed line represents the two-loop EW SM corrections; the solid line represents the two-loop QCD SM corrections. The dotted line represents the full SM two-loop corrections. Finally, the dashed-dotted line represents the corrections due to the full two-loop SM+MW decay width. The MW parameters are chosen as in Fig. 2

$T(R_0) = C(R_0) = 3$, $m_S = 750$ GeV, $\lambda_1(m_S) = 4$, $\lambda_2(m_S) = 1$, $\lambda_3(m_S) = 1/2$, ($\overline{\text{MS}}$ couplings at the scale $\mu = m_S$)[4] and renormalizing the mass of the scalars on-shell. With this set of parameters $A^2/m_0^2 \sim 0.1$, thus it acts as a large suppression factor. The figure shows that the NLO production cross section in this model is always significantly larger that the SM one. In particular, for small values of $m_H$ it is almost two times the SM cross section.

The presence of an additional octet of scalar particle affects not only the production cross section of the standard Higgs boson but also its decay into two photons, that is a very relevant mode for Higgs searches up to $m_H = 140$ GeV. The formulas for the decay width $H \to \gamma\gamma$, including the contribution of colored scalar particles evaluated at the NLO, can be found in Ref. [29].

In Fig. 3 we plot the correction to the decay width $\Gamma(H \to \gamma\gamma)$ originating from the two-loop (NLO) corrections, with respect to the one-loop (LO) SM prediction, assuming the same parameters as in Fig. 2. In the figure, the EW and QCD SM corrections are separately shown as well as their sum. As already pointed out in Refs. [32, 33], the SM NLO EW and QCD corrections basically cancel each other. The effect of the charged scalar particle, on top of the Standard Model particles, is to reduce the LO SM decay width into two photons. At the leading order (one-loop amplitudes), this reduction ranges already

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[4]This set of parameters is consistent with the electroweak precision physics constraints [31]. In this model it is possible to generate a positive contribution to the $\rho$ parameter from the colored scalar sector allowing for an heavier standard Higgs boson in the electroweak fit.

between 9 and 6%. Considering the QCD corrections to the scalar contribution (Fig. 3 dashed-dotted line), the reduction is further increased by an amount that ranges between 13% for $m_H \sim 100$ GeV and 6% for $m_H \sim 160$ GeV. The reduction effect clearly depends on the mass of the scalar particle and can be much more pronounced for smaller values of $m_S$.

It should be recalled that the relevant quantities at LHC for the Higgs discovery are the product of the production cross sections times the branching ratios, so that for $m_H \lesssim 140$ GeV the relevant quantity is $\sigma(pp \to H)BR(H \to \gamma\gamma)$. In this product the reduction effect induced in $H \to \gamma\gamma$ by the scalar contribution is actually more than compensated by the increase in the gluon fusion production cross section, so that the Higgs boson discovery potential at LHC in this model is actually higher than in the SM.

## 3.2   The MSSM

We consider now the contribution of the scalar quarks on the Higgs production cross section in the MSSM. The Higgs sector of the MSSM contains two complex $(\mathbf{1}, \mathbf{2})_{1/2}$, $(\mathbf{1}, \mathbf{2})_{-1/2}$ scalar fields that couple to the down- and up-type fermions separately. After spontaneous symmetry breaking the spectrum of the MSSM Higgs sector contains five physical states, two CP-even neutral boson, $h, H$, one CP-odd neutral one, $A$, and two charged Higgs particles $H^{\pm}$. At the lowest order the MSSM Higgs sector can be specified in terms of two independent parameters, usually chosen as $m_A$, the mass of the pseudoscalar boson, and $\tan\beta = v_2/v_1$, the ratio of the vacuum expectation values of the two Higgs fields.

Stop and sbottom loops can affect significantly the production cross section of the CP-even Higgs bosons. Indeed, there can be regions of the SUSY parameter space in which one of these squarks can be relatively light and its coupling to the $h$ boson relatively strong. As a result this state does not decouple in the $gg \to h$ amplitude and actually its contribution interfere with the fermion one. It should be recalled that when going to the NLO level the purely squark contribution is only part of the MSSM QCD correction to the production cross section. Indeed, at this level, besides diagrams containing quarks or squarks and gluons also diagrams with quark, squark and gluino can contribute and a clear separation between the two contributions is not possible [26]. However, we are going to consider a scenario in which one squark is supposed to be relatively light and therefore to provide the bulk of the corrections while gluino diagrams are supposed to give a small contribution that we are going to neglected. Also we are not going to take into account the quartic self-interaction coupling among squarks.

The computation of $gg \to h/H$ cross section in the MSSM requires the knowledge of the particle mass spectrum of this model. Nowadays, it is available a set of computer codes [35] that allow to compute the entire MSSM spectrum starting from a restricted number of parameters, that can be assigned at a high scale and then evolved down to the weak scale, like for example in a MSUGRA scenario, or directly assigned at the weak scale. At the level of NLO corrections it is important to specify exactly the meaning of these parameters. For what concerns the entries in the squarks mass matrix, the output of these codes is usually expressed in terms of dimensionally reduced $\overline{\mathrm{DR}}$ parameters evaluated at some specified $\mu = \mu_{EWSB}$ scale. Consequently the mass eigenvalues obtained from this squarks mass matrix, as well as the couplings of the squarks to the neutral Higgs bosons should

be intended as $\overline{\text{DR}}$ quantities[5]. Among all the various quantities entering in the formulae for the gluon fusion cross section at NLO only $\mathcal{G}_0^{(2l,C_R)}$ requires an exact specification[6]. In Ref. [29] this quantity is reported in terms of dimensionally regularized $\overline{\text{MS}}$ masses and couplings. Thus, to employ the result of Ref. [29] we have first to convert the $\overline{\text{DR}}$ masses and couplings obtained as output from any $\overline{\text{DR}}$ code into $\overline{\text{MS}}$ quantities. For what concerns the masses one notices that among the various parameters entering in the squark mass matrix

$$m_{\tilde{q}}^2 = \begin{pmatrix} m_{\tilde{q}_L}^2 + m_q^2 + m_z^2(I_q^3 - e_q \sin^2\theta_W)\cos 2\beta & m_q(A_q - \mu\,(\cot\beta)^{2I_q^3}) \\ m_q(A_q - \mu\,(\cot\beta)^{2I_q^3}) & m_{\tilde{q}_R}^2 + m_q^2 + m_z^2\,e_q\sin^2\theta_W\cos 2\beta \end{pmatrix} \quad (41)$$

the soft SUSY breaking left- and right-handed squark masses, $m_{\tilde{q}_L}^2$, $m_{\tilde{q}_R}^2$, the trilinear squark coupling, $A_q$, $\tan\beta$ as well as the Higgs mass parameter $\mu$ at the level of NLO QCD corrections are identical in dimensional regularization and dimensional reduction while the only parameter that requires a conversion is the quark mass as

$$m_q^{(\overline{\text{DR}})} = m_q^{(\overline{\text{MS}})} - \frac{g_s^2}{16\pi^2}C_F\,m_q\;. \quad (42)$$

In Eq. (41) $I_q^3$ is the third component of the weak isospin, $e_q$ the electric charge of the quark $q$, $m_z$ the mass of the $Z$ boson and $\theta_W$ the Weinberg angle. Once an $\overline{\text{MS}}$ squark mass matrix has been constructed one can obtain the $\overline{\text{MS}}$ mass eigenvalues and in case convert them in the OS results. A similar procedure can be employed to obtain the $\overline{\text{MS}}$ couplings of the squarks to the Higgses [36, 25], where also in this case the only quantity that requires a conversion is the quark mass as in Eq. (42).

Having set the framework for the computation of the gluon fusion production cross section of the neutral MSSM Higgs bosons, we consider the particular region of the parameter space, the so-called Higgs gluophobic scenario, in which there is a negative interference between the standard fermionic contribution and the one coming from the stop and sbottom states [27]. As input parameters for the squark mass matrix at the scale $\mu_{EWSB} = 300$ GeV in this scenario we chose $m_{\tilde{q}_L}^2 = m_{\tilde{t}_R}^2 = m_{\tilde{b}_R}^2 = 350$ GeV $A_t = A_b = -600$ GeV, $\mu = 300$ GeV, $m_t^{\overline{\text{MS}}}(\mu_{EWSB}) = 153$ GeV, $m_b^{\overline{\text{MS}}}(\mu_{EWSB}) = 2.3$ GeV.

In Fig. 4 we plot the gluon fusion production cross section for the $h$, $H$ CP-even Higgs bosons at LO and NLO for $\tan\beta = 3$. The $\overline{\text{MS}}$ squark mass eigenvalues are found to be: $m_{\tilde{t}_1} = 190$ GeV, $m_{\tilde{t}_2} = 500$ GeV, $m_{\tilde{b}_1} = 350$ GeV, $m_{\tilde{b}_2} = 360$ GeV while the rest of the MSSM particle spectrum, in particular the masses of the lighter and heavier neutral CP-even Higgs bosons, is obtained using the code Suspect with a gluino mass $m_{\tilde{g}} = 500$ GeV and $M_2 = \mu$. As can be seen from the figure, when the QCD corrections are taken into account the NLO cross section shows an increase comparable to the SM case. Therefore, the QCD corrections to the quark and squark contributions are both large and of similar size. According to the discussion in Section 2 the NLO curve should contain spikes in correspondence of the opening of the $2\,\tilde{t}_{1,2}$, $2\,\tilde{b}_{1,2}$ thresholds. These spikes are actually extremely narrow and either are not drawn or are just hinted in the figure. In Fig. 5 the same analysis is performed for $\tan\beta = 30$ with a corresponding squark mass spectrum

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[5]The codes usually provide also OS masses for the SUSY particles.

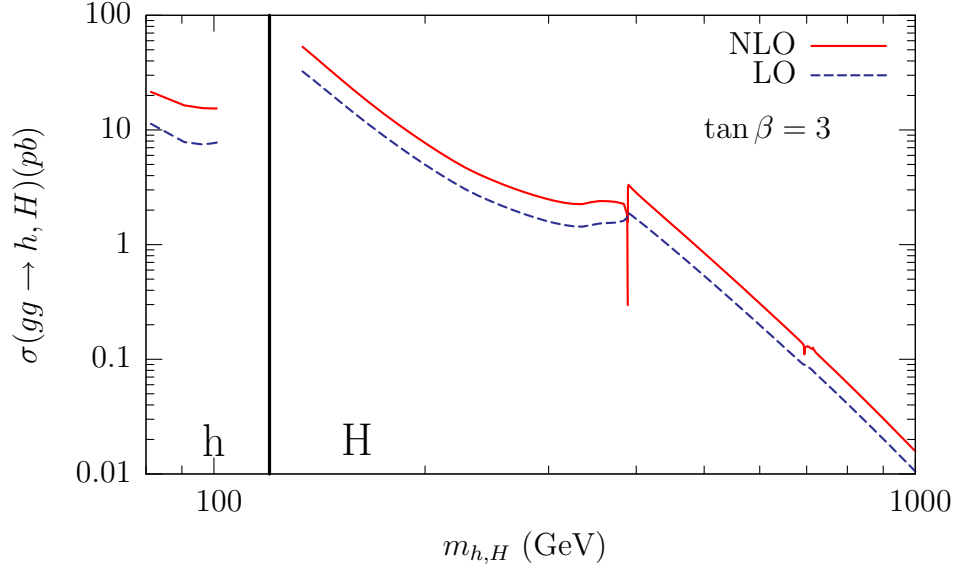[6]For the top and bottom contribution to the gluon fusion cross section we consider always OS masses.

Figure 4: Production cross section of a light (heavy) CP-even Higgs boson, in the MSSM, with $\tan\beta = 3$ at LO (dashed line) and at NLO (solid line).
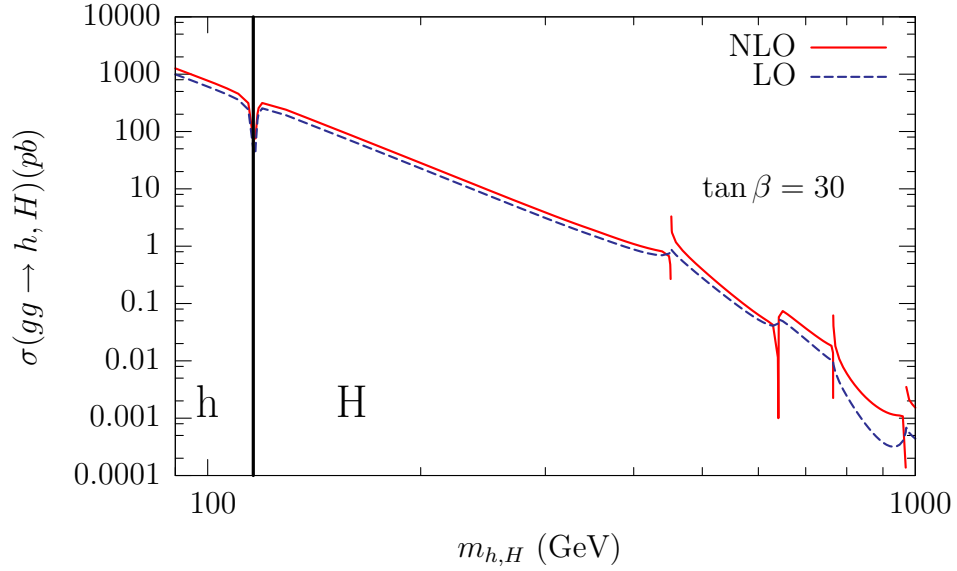


Figure 5: Production cross section of a light (heavy) CP-even Higgs boson, in the MSSM, with $\tan\beta = 30$ at LO (dashed line) and at NLO (solid line).

$m_{\tilde{t}_1} = 230$ GeV, $m_{\tilde{t}_2} = 490$ GeV, $m_{\tilde{b}_1} = 320$ GeV, $m_{\tilde{b}_2} = 380$ GeV. As can be seen from the figure, in this case the NLO corrections are usually percentually smaller than in the SM case. Furthermore, in the singular behaviour at the openings of the squarks thresholds it is possible to appreciate the change of sign when passing through the thresholds due to our choices of $\overline{\text{MS}}$ masses (see Fig. 1). Our results for the MSSM are in agreement with the analysis carried out in Ref. [25].

# 4   Conclusions

In this paper we have presented analytical results for the NLO QCD corrections to the Higgs production cross section in gluon fusion. We considered both the contributions due to colored fermions and colored scalars running in the loops. The analytic formulas are provided in a fully general form and they can be used in computer codes aiming at the phenomenological description of different theoretical models. In particular, the results have been implemented in a `Fortran` code which provides a flexible tool to study BSM physics effects for a generic model which satisfies $SU(N_c) \times SU(2)_L \times U(1)_Y$ gauge invariance.

We have discussed the behaviour of the $K$ factor (the ratio between the NLO and the LO cross section), comparing the exact results with the ones in which the NLO corrections are calculated in the infinite fermion and/or scalar mass approximation. In particular, we recover the well known fact that in the SM the effective theory provides a good approximation of the NLO corrections in a wide range of values of the Higgs mass. The scalar case, however, is more complicated. Actually, a non-physical singularity appears in the two-loop virtual corrections at the scalar pair-production threshold (with different shapes depending on the renormalization scheme in which coupling and masses are renormalized). This alters, to some extent, the discussion about the behaviour of the $K$ factor near the threshold, where the propagator of the scalar field, in principle, should be resummed to all orders in perturbation theory. In the region away from threshold, we see a good convergence to 1 in the light-Higgs region, while a certain deviation from the effective theory shows up for heavy Higgs masses, ranging in any cases within 10%. We notice that, when there is a substantial interference between the fermion and scalar contributions the situation could be more complicated, giving rise to behaviours that could differ substantially from the ones described above.

In the paper, we have applied our results to the study of the Higgs production cross section in two different extensions of the SM: the model proposed by Manohar and Wise, in which the SM is supplemented by an extra colored scalar weak doublet in the adjoint representation of $SU(N_c)$, and the MSSM in the limit of neglecting the gluino contribution. In the MW model, the extra scalars lead to a large enhancement of the Higgs production cross section: with the set of parameters considered, we register an enhancement of up to a factor of 2 with respect to the SM results. In the same model, the Higgs decay width in two photons is decreased by a factor up to 13%. The net effect, considering the combination of production and decay, is a large positive correction. In the MSSM, we consider the so-called gluophobic scenario in which the destructive interference between squark and quark loops reduces significantly the production of the lightest CP-even Higgs boson $h$. In this situation we find that the NLO corrections to the squark contribution are of similar size as those of

the quark part in agreement with previous results in the literature [24, 25].

The study of exclusive observables will be necessary to obtain more realistic phenomenological results: the squared matrix elements described in this paper can be easily embedded in a Monte Carlo code aiming at such studies.

## Acknowledgements

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